RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, JUNE 2022

SECOND YEAR (BATCH 2020-23)

MATHEMATICS (GENERAL)

Paper : IV

Date : 27/06/2022

Time : 11.00 am – 2.00 pm

Full Marks : 75

[Use a separate Answer book for **each group**]

Group – A

Answer **any five** questions of the following: [5×5]

1. Solve: $\left(D^2+4\right)y = x\cos^2 x$, where $D \equiv \frac{d}{dx}$. (5)

2. Solve: $y = px + \frac{a}{p^2}, p = \frac{dy}{dx}$. (5)

3. Solve : $\left(D^2-2D\right)y = e^x \sin x, D \equiv \frac{d}{dx}$ (by the method of variation of parameters). (5)

4. Solve: $xy - \frac{dy}{dx} = e^{-x^2} y^3$. (5)

5. Find $\frac{1}{(D-3)(D-2)}\log_e x$, where $D \equiv \frac{d}{dx}$. (5)

6. Solve : $y_2 + a^2 y = \tan ax$ without using the method of variation of parameters. (5)

7. a) Solve: $y\left(xy + x^2y^2 + 1\right)dx - x\left(x^2y^2 - xy + 1\right)dy = 0.$ (3)

   b) Solve: $xdy - ydx = \left(\sin\frac{1}{x}\right)dx.$ (2)

8. Reduce $e^{3x}(p-1) + p^3 e^{2y} = 0 \left(p = \frac{dy}{dx}\right)$ to Clairaut's form by the substitution $e^x = u, e^y = v.$ Hence solve the equation. (5)

Group – B

Answer **any five** of the following Questions: [5×10]

9. a) State Cauchy's Mean value Theorem.

   b) If $y = \sin\left(m\sin^{-1}x\right)$, prove that $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - \left(n^2-m^2\right)y_n = 0$.

   c) Prove that $\lim_{x\to 0+}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}} = 1$. (2+5+3)

10. a) Find the asymptotes of the curve:

    $\left(x^2-y^2\right)^2 - 8\left(x^2+y^2\right) + 8x - 16 = 0$.

b) Find the envelope of the family of ellipses $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, where the Parameters h and k are connected by $\frac{h^2}{a^2}+\frac{k^2}{b^2}=1$. (5+5)

11. a) Evaluate $\int_0^1 dx\int_x^{\frac{1}{x}}\frac{ydy}{(1+xy)^2(1+y^2)}$, by changing the order of integration.

b) Determine the values of a , b , c so that $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$. (6+4)

12. a) The area enclosed between the are of parabola $y^2=4ax$ from the vertex to one extremity of the latus rectum is revolved about the corresponding chord. Prove that the volume of the spindle so formed is $\frac{2\sqrt{5}}{75}\pi a^3$.

b) Find the area of surface obtained by revolving the parabola $y^2=4ax$ about $x-axis$ (The segment from vertex to the point with the abscissa $x=3a$) . (5+5)

13. a) Show that $\int_0^\infty e^{-x^4}dx\times\int_0^\infty x^2e^{-x^4}dx=\frac{\pi}{8\sqrt{2}}$.

b) For $m>-1, n>-1$, prove that $\int_a^b (x-a)^m(b-x)^n dx=(b-a)^{m+n-1}\frac{\Gamma(m+1)\Gamma(n+1)}{\Gamma(m+n+2)}$. (4+6)

14. a) Show that the limit $\lim_{(x,y)\to(0,0)}\frac{x^3+y^3}{x-y}$ does not exist.

b) Using the definition, prove that $\lim_{(x,y)\to(0,0)}\frac{x^4+y^4}{x^2+y^2}=0$. (5+5)

15. a) If E is the region bounded by the circle $x^2+y^2-2ax-2by=0$, evaluate $\iint_E \sqrt{x(2a-x)+y(2b-y)}\ dxdy$.

b) If E is the region bounded by the circle $x^2+y^2-2ax=0$, evaluate $\iint_E \sqrt{4a^2-x^2-y^2}\ dxdy$. (5+5)

16. a) A quadrant of a circle $x^2+y^2=a^2$ is revolved about the chord joining the point of intersection of circle and the axes. Find the volume of the spindle generated.

b) Show that $\int_0^{\pi/2}(\sin^p x)dx\times\int_0^{\pi/2}\left(\sin^{(p+1)} x\right)dx=\frac{\pi}{2(p+1)}$. (6+4)

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